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Semiactive Vibration Control of Train Suspension Systems via Magnetorheological Dampers

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ABSTRACT: This paper is aimed to show the feasibility for improving the ride quality of railway vehicles with semiactive secondary suspension systems using magnetorheological (MR) dampers. A nine degree-of-freedom railway vehicle model, which includes a car body, two trucks and four wheelsets, is proposed to cope with vertical, pitch and roll motions of the car body and trucks. The governing equations of the railway vehicle suspension systems integrated with MR dampers are developed. To illustrate the feasibility and effectiveness of the controlled MR dampers on railway vehicle suspension systems, the LQG control law using the acceleration feedback is adopted as the system controller, in which the state variables are estimated from the measurable accelerations with the Kalman estimator. In order to make the MR dampers track the optimal damping forces, a damper controller to command the voltage to the current drivers for the MR dampers is proposed. The acceleration responses of the car body of the train vehicle with semiactive secondary suspension system integrated with MR dampers are evaluated under random and periodical track irregularities. This semiactive controlled system is also compared to the conventional passive suspension system using viscous dampers without MR dampers, and the secondary suspension system integrated with MR dampers in passive on and passive off modes. The simulation results show that the vibration control of the train suspension system with semiactive controlled MR dampers is feasible and effective.

Key Words: train suspension, magnetorheological damper, vibration control, semiactive control.

INTRODUCTION

The development of high-speed railway vehicles has been a great interest of many countries because high-speed trains have been proven as an efficient and economical transportation means while minimizing air pollution. However, the high speed of the train would cause significant car body vibrations, which induce the following problems: the ride stability, the ride quality, and the cost of track maintenance. Thus the vibration control of the car body is needed to improve the ride comfort and safety of a train. Various kinds of railway vehicle suspensions linking the bogies and the car bodies have been designed to cushion riders from vibrations. In general, the suspension systems used in railway vehicles can be categorized as passive, active, and semiactive types. A passive railway vehicle suspension employing springs and pneumatic or oil dampers has some advantages such as design simplicity and cost effectiveness. However the performance on the wide frequency range could be limited. Therefore, several researchers (Sasaki et al., 1994; Shimamune and Tanifuji, 1995) have proposed and investigated active suspension technology for railway vehicles, which utilize oil cylinders and pneumatic actuators. The active suspension provides high control performance over wide frequency range, but it requires high power and sophisticated control implementation. Furthermore, the active control suspension system would import the mechanical power into the system, so the stability of the control system needs to be considered. In recent years, semiactive suspension systems that utilize controllable devices based on smart fluids have drawn the attention of many researchers. The essential characteristics of the smart fluids are their abilities to reversibly change from a free-flowing fluid to a semisolid with controllable yield strength in milliseconds when exposed to an electric or magnetic field (Sims et al., 1999). Two fluids that are viable contenders for the development of controllable dampers are electrorheological (ER) and magnetorheological (MR) fluids (Wereley and Pang, 1998).

Peel et al. (1996) have developed a mathematical model of a controllable vibration damper employing ER fluids intended for the application to suspension systems...
of ground vehicles. The modeling technique was illustrated in an application for controlling the lateral dynamics of a modern rail vehicle. However, it is well known that ER fluids are excited by high electric fields. To produce sufficient levels of field strength requires high voltage, which restrains its potential applications due to the safety problems. On the other hand, MR fluids are excited by a magnetic field, which can be generated by a low voltage source. In addition, MR fluids generate significantly larger dynamic force levels than ER fluids and operate over wide temperature ranges. More recently, the semiactive dampers using MR fluids are developed and applied to control the vibration of automobiles and heavy trucks by some researchers (Choi et al., 2000; Simon and Ahmadian, 2001). In this paper, a semiactive secondary suspension system with MR dampers for a full-size railway vehicle is modeled and the system performance is evaluated.

RAILWAY VEHICLE SUSPENSION SYSTEMS

In this study, a full-size train vehicle model has been formulated for a railway vehicle running on a straight track. A nine degree-of-freedom analytical model is considered. This model, which is shown in Figure 1, consists of a car body, two truck frames, and four wheelsets. The wheelsets and truck frames are connected by a primary suspension system that consists of springs and viscous dampers. The car body and truck frames are connected by springs and MR dampers (in vertical direction, denoted as “MRD” in Figure 1)/viscous dampers (in lateral direction, denoted as “PD” in Figure 1), which are referred to as the secondary suspension system. For the semiactive train suspension system in this study, only four MR dampers are included by replacing the vertical viscous dampers between the car body and two trucks. The schematic configuration of the semiactive control system for the railway vehicle is shown in Figure 2. In this system, four MR dampers, which are used to control the vertical, pitch and roll vibrations, are vertically placed on the left and right sides of each truck (denoted as MRD_{zlr}, MRD_{zll}, MRD_{ztr}, and MRD_{ztl} respectively in Figure 2). In order to realize the feedback control, the accelerations of the car body and trucks are measured with accelerometers, which are installed in vertical and lateral directions as shown in Figure 2.

Figure 1. Nine degree-of-freedom train vehicle model with MR dampers.

Figure 2. Schematic of semiactive control system for railway vehicle.
Analytical Model of Railway Vehicle

The governing equations of motion for the railway vehicle with suspension systems can be derived using Newton’s Laws and the nomenclatures used in the development of the formulation are defined in APPENDIX I. The governing equations for the car body (vertical $z$, pitch $\psi$, and roll $\theta$) can be expressed as

$$m_c \ddot{z}_c = F_{szl} + F_{szl} + F_{szl} + F_{szl} + f_{szl} + f_{szl} + f_{szl}$$

where $F$ with different subscripts in Equations (1)–(9) represent the suspension forces produced by the secondary and primary suspensions and the definitions of $F$ in Equations (1)–(9) are listed in APPENDIX II. $F_{szl}, f_{szl}, f_{szl}$, and $f_{szl}$ represent the damping forces produced by the corresponding MR dampers $MRD_{szl}, MRD_{szl}, MRD_{szl}$, and $MRD_{szl}$, which are illustrated in Figure 2. In the subscripts of $F$ and $f$, the first letter (“s” or “p”) is used to represent the secondary and primary suspensions respectively (“s” – secondary suspension, “p” – primary suspension). The second letter of the subscripts (“t”, “i”, “1–4”) of the subscripts is used to identify the trucks (“t” – the leading truck, “i” – the trailing truck) and the wheelsets (“1–4” – the wheelsets 1–4 respectively as marked in Figure 1). The last letter of the subscripts (“r”, “y”) is used to identify the side of suspension system for the corresponding forces (“r” – the parts located on the right side, “y” – the parts located on the left side of the car body, the trucks, and the wheelsets). It should be noted that the secondary vertical damping $c_{sz}$ is zero for the system with MR dampers since the corresponding viscous dampers have been replaced by the MR dampers. However, for the purpose of comparisons, the model developed here can also be used for analyzing the conventional passive system, which the secondary vertical viscous dampers instead of MR dampers are used. In this case, those damping forces $f_{szl}, f_{szl}, f_{szl}$, and $f_{szl}$ corresponding to MR dampers are zero.

MR Damper Model

The MR damper model proposed by Spencer et al. as shown in Figure 3 is adopted in this study. The phenomenological model is governed by the following equations (Spencer Jr. et al., 1997)

$$f = c_1 \dot{y} + k_1(x - x_0)$$

where $F$ with different subscripts in Equations (1)–(9) represent the suspension forces produced by the secondary and primary suspensions and the definitions of $F$ in Equations (1)–(9) are listed in APPENDIX II. $F_{szl}, f_{szl}, f_{szl}$, and $f_{szl}$ represent the damping forces produced by the corresponding MR dampers $MRD_{szl}, MRD_{szl}, MRD_{szl}$, and $MRD_{szl}$, which are illustrated in Figure 2. In the subscripts of $F$ and $f$, the first letter (“s” or “p”) is used to represent the secondary and primary suspensions respectively (“s” – secondary suspension, “p” – primary suspension). The second letter of the subscripts (“t”, “i”, “1–4”) of the subscripts is used to identify the trucks (“t” – the leading truck, “i” – the trailing truck) and the wheelsets (“1–4” – the wheelsets 1–4 respectively as marked in Figure 1). The last letter of the subscripts (“r”, “y”) is used to identify the side of suspension system for the corresponding forces (“r” – the parts located on the right side, “y” – the parts located on the left side of the car body, the trucks, and the wheelsets). It should be noted that the secondary vertical damping $c_{sz}$ is zero for the system with MR dampers since the corresponding viscous dampers have been replaced by the MR dampers. However, for the purpose of comparisons, the model developed here can also be used for analyzing the conventional passive system, which the secondary vertical viscous dampers instead of MR dampers are used. In this case, those damping forces $f_{szl}, f_{szl}, f_{szl}$, and $f_{szl}$ corresponding to MR dampers are zero.

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$$f = c_1 \dot{y} + k_1(x - x_0)$$

$$z = -\gamma |x - \dot{y}|z^{n-1} z - \beta (\dot{x} - \dot{y}) |z|^n + A(x - \dot{y})$$

$$\dot{y} = \frac{1}{c_0 + c_1} [\alpha z + c_0 \dot{x} + k_0(x - y)]$$

Figure 3. Mechanical model for MR damper.
The elements $z_{i2}$ and $z_{i1}$ are the disturbance vectors on the wheels of the train vehicle, which are shown in Figure 1. $F_D$ and $F_e$ are the coefficient matrices of the damping force and the disturbance vector.

By defining the state vector as $x = [q]$, the governing Equation (15) can be rewritten in the state-space form as follows

$$
\frac{dx}{dt} = Ax + Bu(t) + Gw(t)
$$

$$
y = C_0x + Du(t) + Hw(t) + v
$$

where

$$
A = \begin{bmatrix} 0 & 1 \\ \end{bmatrix}, \quad A \in \mathbb{R}^{18x18};
$$

$$
B = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}F_u \end{bmatrix},
$$

$$
B \in \mathbb{R}^{18x4}; \quad G = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}F_w \end{bmatrix},
$$

$$
G \in \mathbb{R}^{18x16};
$$

$$
C_0 = \begin{bmatrix} \mathbf{M}^{-1}K & \mathbf{M}^{-1}C \end{bmatrix}, \quad C_0 \in \mathbb{R}^{9x18};
$$

$$
D = \mathbf{M}^{-1}F_u, \quad D \in \mathbb{R}^{9x4};
$$

$$
H = \mathbf{M}^{-1}F_w, \quad H \in \mathbb{R}^{9x16};
$$

$y$ is the output vector; $v$ is the sensor noise vector.

**SEMIACTIVE CONTROLLER DESIGN**

In order to evaluate the effectiveness of the semiactive suspension system with MR dampers, the Linear Quadratic Gaussian (LQG) control algorithm is employed. The performance index is chosen as

$$
J = \lim_{t_0 \to \infty} \frac{1}{T} \mathbb{E} \left\{ \int_{t_0}^{t_0+T} [x^T(t)Qx(t) + u_d(t)^T R u_d(t)]dt \right\}
$$

where $Q$ and $R$ are symmetric semipositive-definite and positive-definite matrices. $u_d(t)$ is the vector of desired damping forces of the MR dampers, and $u_d(t) = [f_{szl} f_{szl} f_{szl} f_{szl}]^T$. The control law that minimizes Equation (17) is given by

$$
u_d(t) = -\mathbf{K}\hat{x}(t)
$$

where $\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T \mathbf{S}$, $\mathbf{S}$ is determined by $\mathbf{SBR}^{-1}\mathbf{B}^T \mathbf{S} = \mathbf{SA} - \mathbf{A}^T \mathbf{S} = \mathbf{Q}$, $\hat{x}(t)$ is obtained from the following Kalman estimator (Siouris, 1996)

$$
\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_f[y(t) - C_0\hat{x}(t) - Du(t)]
$$

where

$$
K_f = S_f C_f R_f^{-1}, \quad S_f = \mathbb{S}_f C_f R_f^{-1} C_s S_f - \mathbb{A}^T S_f = GQ_f G^T. \quad \mathbb{Q}_f = E(ww^T) \quad \text{and} \quad R_f = E(vv^T)
$$

$\mathbb{Q}_f$ and $\mathbb{R}_f$ are symmetric, semipositive-definite and positive-definite respectively.

The desired damping forces vector $u_d(t)$ of the MR dampers can be obtained from the system controller.
and the desired damping force is set by

\begin{equation}
\begin{aligned}
V & = \frac{V_{\text{max}}}{2N} \sum_{0 \leq t \leq N-1} \left\{ \text{sgn} \left[ f^d - (1 - ki) f \right] + 1 \right\} \\
\end{aligned}
\end{equation}

where \(\text{sgn}(\cdot)\) is the signum function; \(f^d\) and \(f\) are the desired damping force and the actual damping force of the MR damper, respectively; \(N\) is a positive integer and \(0 \leq t \leq N - 1; k\) is a small constant; \(V_{\text{max}}\) is the maximum input voltage to the current driver for the MR damper. To determine the command voltage according to Equation (20), \(N\) times of comparisons between the desired and the actual damping forces are needed. For the \(i\)th comparison, if \(|f^d - (1 - ki) f| > 0\), \(f^d\) and \(f\) have the same sign as well, then the contribution of this comparison to the command voltage will be \(v_i = V_{\text{max}}/2N \times 2\); otherwise the contribution to the command voltage will be \(v_i = V_{\text{max}}/2N \times 0\). All the comparisons for \((i = 0, 1, \ldots, N-1)\) are carried out simultaneously and the command voltage is the summation of the contribution of \(N\) comparisons to the command voltage, that is \(v = \sum_{0 \leq i \leq N-1} v_i\). So \(N\) determines the number of comparisons between the desired and the actual damping forces in order to set the command voltage. In this study, \(N = 6\), \(k = 5.00 \times 10^{-4}\), and \(V_{\text{max}} = 12V\) are used.

To illustrate the effectiveness of the MR damper controller (Equation (20)), an example is considered here. The displacement input across the damper is given by

\begin{equation}
x = 2 \sin \left( 40t - \frac{\pi}{3} \right)
\end{equation}

and the desired damping force is set by

\begin{equation}
f^d = 3600 \sin(40t)
\end{equation}

Then the command Equation (20) is used to let the MR damper track the desired damping force determined by Equation (22). Figure 4 shows the time history of the controlled damping force and the corresponding command voltage, while the damping forces of other cases are also shown for the purpose of comparison. It can be seen that the damping forces in passive on mode (constant command voltage, \(v = 12V\)) and passive off mode (constant command voltage, \(v = 0V\)) have significant deviations from the desired damping force. On the other hand, the controlled damping force tracks well the desired one by applying voltage input as Equation (20).

**SIMULATION RESULTS AND DISCUSSIONS**

In the simulation, the elements of the weighting matrix \(Q(i, j) = 0\) for \(i = 1 \sim 18; j = 1 \sim 18\) except \(Q(1, 1) = 8 \times 10^8; Q(2, 2) = 8 \times 10^8; Q(3, 3) = 8 \times 10^8; Q(13, 13) = 100; Q(16, 16) = 100\) and the weighting matrices \(R = 0.01I_{4 \times 4}; Q_f = 20I_{16 \times 16}; R_f = I_{9 \times 9}\) (\(I\) is the identity matrix, and their subscripts represent the corresponding dimensions). The parameters of a railway vehicle are given in Table 2 and the parameters of the MR damper model are shown in Table 1. It is assumed that the wheels of the vehicle system follow the rails perfectly. Two track irregularities are considered here: one is the random track irregularity and the other is the periodical track irregularity.

The power spectrum densities (PSD) of vertical, pitch and roll accelerations of the car body of the train vehicle under random track irregularities are illustrated in Figure 5. The RMS accelerations of the car body are shown in Table 3. The “Passive” case represents the conventional passive system using viscous dampers without MR dampers for the secondary suspension system. Using four MR dampers by replacing the vertical viscous dampers for the secondary suspension system, there are three cases. When the command voltage to the current drivers for the MR dampers is zero or constant 12 V, they are referred as “Passive Off” and “Passive On”, respectively. For the “Semiactive Control” case, the MR dampers are operated in semiactive control mode via the system and damper controllers.
To evaluate the ride quality, four positions in the passenger compartment are chosen. The first two positions are in the front car body at \( (x_d, y_d, z_d) \) and \( (x_{d'}, \ y_{d'}, \ z_{d'}) \), and the other two positions are in the rear car body at \( (-x_d, y_d, z_d) \) and \( (-x_{d'}, \ y_{d'}, \ z_{d'}) \), where are shown in Figure 1. The accelerations at these four passenger points of the train vehicle are given by

\[
\ddot{z}_p(x_d, y_d, z_d) = \ddot{z}_c - x_d \ddot{\phi}_c + y_d \ddot{\theta}_c \\
\ddot{z}_p(x_{d'}, y_{d'}, z_{d'}) = \ddot{z}_c - x_{d'} \ddot{\phi}_c - y_{d'} \ddot{\theta}_c
\]

\( \text{Table 2. Parameters for railway vehicle model.} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_c )</td>
<td>( 3.96 \times 10^4 ) kg</td>
<td>( m_t )</td>
<td>( 3.25 \times 10^3 ) kg</td>
<td>( V )</td>
<td>55.56 m/s</td>
</tr>
<tr>
<td>( l_{cx} )</td>
<td>( 8.85 \times 10^6 ) kg m^2</td>
<td>( l_{cy} )</td>
<td>( 2.46 \times 10^6 ) kg m^2</td>
<td>( I_{cz} )</td>
<td>( 2.505 \times 10^6 ) kg m^2</td>
</tr>
<tr>
<td>( l_{cz} )</td>
<td>( 3.06 \times 10^5 ) kg m^2</td>
<td>( l_{cz} )</td>
<td>( 3.02 \times 10^5 ) kg m^2</td>
<td>( k_{cz} )</td>
<td>( 4.27 \times 10^3 ) N/m</td>
</tr>
<tr>
<td>( c_{pz} )</td>
<td>( 4.00 \times 10^6 ) N/m</td>
<td>( k_{pz} )</td>
<td>( 3.25 \times 10^5 ) N/m</td>
<td>( k_{px} )</td>
<td>( 7.00 \times 10^5 ) N/m</td>
</tr>
<tr>
<td>( c_{pz} )</td>
<td>( 1.5 \times 10^5 ) N/m</td>
<td>( k_{pz} )</td>
<td>( 1.50 \times 10^5 ) N/m</td>
<td>( k_{px} )</td>
<td>( 7.00 \times 10^5 ) N/m</td>
</tr>
<tr>
<td>( k_{cx} )</td>
<td>( 2.0 \times 10^5 ) N/m</td>
<td>( k_{cy} )</td>
<td>( 5.00 \times 10^4 ) N/m</td>
<td>( k_{cz} )</td>
<td>( 8.00 \times 10^4 ) N/m</td>
</tr>
<tr>
<td>( c_{cx} )</td>
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<td>( k_{cz} )</td>
<td>( 2.90 \times 10^6 ) N/m</td>
<td>( k_{px} )</td>
<td>( 7.00 \times 10^5 ) N/m</td>
</tr>
<tr>
<td>( h_{ts} )</td>
<td>( 0.217 ) m</td>
<td>( h_{cs} )</td>
<td>( 1.207 ) m</td>
<td>( h_{tp} )</td>
<td>( -0.452 ) m</td>
</tr>
<tr>
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<td>( l )</td>
<td>( 9.00 ) m</td>
<td>( a )</td>
<td>( 0.7465 ) m</td>
</tr>
<tr>
<td>( \delta_s )</td>
<td>( 1.00 ) m</td>
<td>( b )</td>
<td>( 1.25 ) m</td>
<td>( \delta_p )</td>
<td>( 1.00 ) m</td>
</tr>
</tbody>
</table>

*The value of \( c_{sz} \) is used for the conventional passive secondary suspension system in order to compare with the semiactive secondary suspension. For the secondary suspension integrated with MR dampers, \( c_{sz} = 0 \).
\[ \ddot{z}_{fr}(x_d, y_d, z_d) = \ddot{z} + x_d\ddot{\phi}_r + y_d\ddot{\theta}_r \quad (23c) \]

\[ \ddot{z}_{rl}(x_d, y_d, z_d) = \ddot{z} + x_d\ddot{\phi}_l - y_d\ddot{\theta}_l \quad (23d) \]

Figures 6 and 7 show the PSD and time histories of the acceleration responses \( \ddot{z}_{fr} \) and \( \ddot{z}_{rl} \) at two passenger points in the car body, where \( x_d = 9 \text{ m}, y_d = 0.75 \text{ m}, \) and \( z_d = -0.2 \text{ m}. \) The \( \ddot{z}_{fr} \) is the acceleration on the right side of the front car body, and the \( \ddot{z}_{rl} \) is the acceleration on the left side of the rear car body. The RMS accelerations at four passenger points in the car body are also given in Table 3.

Observing Figure 5(a) and (b), those passive and semiactive cases can be compared. For the first peak frequency responses of the car body vertical and pitch accelerations, the conventional passive secondary suspension system has better isolation than the “Passive Off” system, in which the MR dampers are not activated (0 V). On the other hand, for attenuating the frequency responses other than the first resonance, the secondary suspension system integrated with MR dampers in the passive off mode is superior to the conventional passive suspension system. For the “Passive On” case (constant 12 V applied to the MR dampers), the car body vertical and pitch accelerations lie between the “Passive” and “Passive Off” cases for most of the frequencies shown. While the MR dampers are operated in semiactive control mode, it can be seen that the car body accelerations are significantly reduced compared to those three passive cases.

From Figure 5(c), it is illustrated that the secondary suspension system integrated with MR dampers (no matter whatever modes are employed) is especially effective for reducing the roll vibration of the car body compared to the conventional passive suspension system. The results can also be quantified from Table 3. While the vertically installed MR dampers are effective for controlling the vertical vibration of the car body (about 29% improvement in terms of vibration reduction for “Semiactive Control” vs. “Passive On”), it is even more significant for the controlled MR dampers for reducing the pitch and roll vibrations (41 and 57%, respectively).

From Figures 6 and 7, the accelerations at the passenger points are also significantly attenuated through the semiactive secondary suspension system.

| Table 3. Car body RMS acceleration under random excitation. |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                                | Passive \(( \times 10^{-3} )\) | Passive Off \(( \times 10^{-3} )\) | Passive On \(( \times 10^{-3} )\) | Semiactive Control \(( \times 10^{-3} )\) |
|--------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Vertical acceleration \(( \text{m/s}^2 )\) | 6.9             | 5.8             | 3.1             | 2.2             | 68.12           | 62.07           | 29.03           |
| Pitch acceleration \(( \text{rad/s}^2 )\)  | 1.3             | 1.3             | 0.566           | 0.336           | 74.15           | 74.15           | 41.07           |
| Roll acceleration \(( \text{rad/s}^2 )\)   | 2.1             | 0.462           | 0.388           | 0.167           | 91.62           | 63.88           | 56.96           |
| Acceleration at Passenger Points \(( \text{m/s}^2 )\) | 13.9            | 10.7            | 5.0             | 3.0             | 78.42           | 71.96           | 40.00           |
| \( \ddot{z}_{fr} \)             | 13.9            | 10.7            | 5.0             | 3.0             | 78.42           | 71.96           | 40.00           |
| \( \ddot{z}_{rl} \)             | 14.2            | 14.8            | 5.9             | 3.9             | 72.54           | 73.65           | 33.90           |
| \( \ddot{z}_{rr} \)             | 14.2            | 14.8            | 5.9             | 3.9             | 72.54           | 73.65           | 33.90           |

**Figure 6.** Acceleration response \( \ddot{z}_x \) at \((x_d,y_d,z_d)\) under random track irregularities: (a) PSD; (b) Time history.
compared to those through the conventional passive suspension system when the train vehicle experiences the random track irregularities. It is clearly shown that the reduction of the accelerations at the passenger points through semiactive control is also improved compared to those with constant 12 voltage (Passive On) and 0 voltage (Passive Off) to the MR dampers, when the secondary suspension system is integrated with MR dampers. The performance can also be verified from Table 3. It should be noted that the vertical accelerations at passenger points (34% or 40% reductions for “Semiactive Control” versus “Passive On”) can be reduced more than that of the car body (29%) due to the coupling effects among the vertical, pitch and roll motions.

In Figure 8, the damping forces and the corresponding command voltage for the MR damper MRD\textsubscript{szlr} under random track irregularities are plotted. It can be seen that the controlled damping force of the MR damper can follow the desired damping force on the whole. When the MR damper is employed in passive on or passive off modes, the MR damping force has more derivation from the desired damping force. Even through the controlled damping force cannot perfectly follow the desired damping force, the semiactive control is still superior to the “Passive On” and “Passive Off” cases with the MR dampers.

If the vertical excitation of the front wheelset (the wheelset 1 as indicated in Figure 1) of the leading truck is represented by $z_1$ and $z_2$, the periodic track irregularities can be expressed as

$$z_1(t) = \frac{4A}{\pi} \left[ \frac{1}{3} \cos \Omega x - \frac{1}{15} \cos 2\Omega x + \frac{1}{35} \cos 3\Omega x \right],$$

$$z_2(t) = \frac{4A}{\pi} \left[ \frac{1}{3} \cos \Omega y - \frac{1}{15} \cos 2\Omega y + \frac{1}{35} \cos 3\Omega y \right].$$

(24)

where $A$ is the scalar factor of the periodic irregularities of the track, $\Omega (\Omega = 2\pi/L$, unit: rad/m) is the spatial frequency, $L$ is the spatial length (the rail length), and $x = Vt$. In this paper, $A = 25.4$ and $L = 25$ m. In addition, the vertical excitations to other wheelsets $z_3 \sim z_4, z_5 \sim z_6$ are described by

$$z_3(t) = \frac{4A}{\pi} \left[ \frac{1}{3} \cos \Omega z - \frac{1}{15} \cos 2\Omega z + \frac{1}{35} \cos 3\Omega z \right],$$

$$z_4(t) = \frac{4A}{\pi} \left[ \frac{1}{3} \cos \Omega y - \frac{1}{15} \cos 2\Omega y + \frac{1}{35} \cos 3\Omega y \right].$$

(25)

From Equations (24) and (25), the disturbance vector $w$ in Equation (15) can be determined. Figure 9 shows the time responses of the car body accelerations under the given periodical track.
irregularities. Almost the same levels of the car body accelerations are obtained for the systems with conventional passive viscous dampers ("Passive") and "Passive Off" MR dampers. Under the periodical excitations, the accelerations of the car body when MR dampers are employed in passive on mode are larger than those in passive off mode. This shows that the increase in the damping forces is not always beneficial to the vibration attenuation when the MR dampers are integrated into the system. On the other hand, the controlled accelerations of the car body in the vertical and pitch motions are greatly reduced (about 50%) compared to the passive on case.

However, the roll accelerations of the car body are found to be very small compared to the pitch ones. It is because the given periodical track irregularities do not include cross level track irregularities. While asynchronous track irregularities are considered, the control effects on the roll motion of the car body would be more significant as shown in Figure 5(c) under the random excitation.

The damping forces and the corresponding command voltage for the MR damper $MRD_{zlr}$ under periodical track irregularities are also shown in Figure 10. It can be seen again that the controlled damping force of the MR damper can follow the desired damping force on the whole under periodical track irregularities. When the MR damper is employed in passive on or passive off modes, the damping force of the MR damper has more deviation from the desired damping force. Therefore, it can be concluded again that the semiactive control is superior to the passive on and passive off modes of the MR damper under periodical track irregularities, even the controlled damping force cannot perfectly follow the desired damping force.

**CONCLUSION**

In this paper, a semiactive secondary train suspension system with MR dampers has been investigated by considering a full-size railway vehicle, which includes three vibration motions (vertical, pitch and roll) of the car body and trucks. The governing equations of a nine degree-of-freedom railway vehicle model integrated with MR dampers are developed. To illustrate the feasibility and effectiveness of controlled MR dampers on railway vehicle suspension systems, the LQG control using the acceleration feedback is adopted as the system controller, in which the state variables are estimated from the measurable accelerations with the Kalman estimator. In order to let the MR dampers track the optimal damping forces, a signum function based control algorithm to command the voltage to the current drivers for MR dampers is proposed (damper controller). The acceleration responses of the car body and four passenger points of the train vehicle with semiactive
secondary MR suspension system and with conventional passive secondary suspension system using viscous dampers under random and periodical track irregularities are evaluated and compared with each other. For the secondary suspension system integrated with MR dampers, the other two cases, in which the MR dampers are employed in passive on or passive off modes, are also investigated.

The simulation results show that the semiactive secondary suspension system can significantly attenuate the vibrations of the car body and is particularly effective for reducing the roll vibration of the car body of the railway vehicle under random track irregularities. The results also show that the increase of damping forces of MR dampers is not always beneficial to attenuate vibration of the vehicle car body. When MR dampers are integrated into the suspension system of the vehicle, the optimal damping forces produced by the MR dampers can be obtained through suitable system and damper controllers. In summary, the results of this study show that the vibration control of the train suspension system with semiactive controlled MR dampers is feasible and effective. However, this paper only presents some preliminary research results that verify the feasibility of applying MR dampers to train suspension systems, more thorough research work is under investigation.

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APPENDIX I: NOMENCLATURE

Vehicle system

$k_{xx} =$ secondary longitudinal stiffness
$k_{xy} =$ secondary lateral stiffness
$k_{xz} =$ secondary vertical stiffness
$c_{xx} =$ secondary longitudinal damping
$c_{xy} =$ secondary lateral damping
$c_{xz} =$ secondary vertical damping

$l =$ half of truck center pin spacing
$b =$ half of wheelbase
$d_p =$ half of primary suspension spacing (lateral)
$a =$ half of wheelset contact distance
$d_s =$ half of secondary suspension spacing (lateral)
$h_{ts} =$ vertical distance from truck frame center of gravity to secondary suspension
$h_{cs} =$ vertical distance from car body center of gravity to secondary suspension
$h_{sp} =$ vertical distance from truck frame center of gravity to primary suspension
$h_{yp} =$ vertical distance from wheelset center of gravity to primary suspension
$V =$ velocity of railway vehicle

Rail track irregularities

$z_{1r} =$ vertical disturbance acted on the right wheel of wheelset 1
$z_{1l} =$ vertical disturbance acted on the left wheel of wheelset 1
$z_{2r} =$ vertical disturbance acted on the right wheel of wheelset 2
$z_{2l} =$ vertical disturbance acted on the left wheel of wheelset 2
$z_{3r} =$ vertical disturbance acted on the right wheel of wheelset 3
$z_{3l} =$ vertical disturbance acted on the left wheel of wheelset 3
$z_{4r} =$ vertical disturbance acted on the right wheel of wheelset 4
$z_{4l} =$ vertical disturbance acted on the left wheel of wheelset 4

MR Damper

$c_0 =$ viscous damping observed at large velocities
$c_{0a}, c_{0b} =$ constants that determine $c_0$
$k_0 =$ stiffness at large velocities
$c_1 =$ viscous damping observed at low velocities
$c_{1a}, c_{1b} =$ constants that determine $c_1$
$k_1 =$ accumulator stiffness
$x_0 =$ initial displacement of spring $k_1$ associated with nominal damper force due to accumulator
$\alpha =$ scaling value for the Bouc-Wen model
$\alpha_a, \alpha_b =$ constants that determine $\alpha$
$\gamma, \beta, A, n =$ parameters that determine the hysteresis loop in Bouc-Wen model
$\eta =$ constant to govern the first-order filter
APPENDIX II: DEFINITIONS OF $F$ IN EQUATIONS (1)–(9)

\[
\begin{align*}
F_{xlb} &= -k_{s}(c_{x}q_{c} + h_{s}q_{1}) - c_{s}(h_{s}q_{c} + h_{s}q_{1}); \\
F_{xll} &= -k_{s}(c_{x}q_{c} + h_{s}q_{1}) - c_{s}(h_{s}q_{c} + h_{s}q_{1}); \\
F_{xlr} &= -k_{s}(c_{x}q_{c} + h_{s}q_{2}) - c_{s}(h_{s}q_{c} + h_{s}q_{2}); \\
F_{xll} &= -k_{s}(c_{x}q_{c} + h_{s}q_{2}) - c_{s}(h_{s}q_{c} + h_{s}q_{2}); \\
F_{ybr} &= k_{y}(c_{y}q_{c} + h_{y}q_{1}) + c_{y}(h_{y}q_{c} + h_{y}q_{1}); \\
F_{yll} &= k_{y}(c_{y}q_{c} + h_{y}q_{1}) + c_{y}(h_{y}q_{c} + h_{y}q_{1}); \\
F_{yrl} &= k_{y}(c_{y}q_{c} + h_{y}q_{2}) + c_{y}(h_{y}q_{c} + h_{y}q_{2}); \\
F_{yll} &= k_{y}(c_{y}q_{c} + h_{y}q_{2}) + c_{y}(h_{y}q_{c} + h_{y}q_{2}); \\
F_{szbr} &= -k_{c}[z_{c} - l_{c}q_{c} - z_{1} + d_{c}(q_{c} - q_{1})] \\
&\quad - c_{z}[z_{c} - l_{c}q_{c} - z_{1} + d_{c}(q_{c} - q_{1})]; \\
F_{szll} &= -k_{c}[z_{c} - l_{c}q_{c} - z_{1} + d_{c}(q_{c} - q_{1})] \\
&\quad - c_{z}[z_{c} - l_{c}q_{c} - z_{1} + d_{c}(q_{c} - q_{1})]; \\
F_{szrl} &= -k_{c}[z_{c} + l_{c}q_{c} + z_{2} + d_{c}(q_{c} - q_{2})] \\
&\quad - c_{z}[z_{c} + l_{c}q_{c} + z_{2} + d_{c}(q_{c} - q_{2})]; \\
F_{szll} &= -k_{c}[z_{c} + l_{c}q_{c} + z_{2} + d_{c}(q_{c} - q_{2})] \\
&\quad - c_{z}[z_{c} + l_{c}q_{c} + z_{2} + d_{c}(q_{c} - q_{2})]; \\
F_{pxbr} &= -k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pxll} &= -k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pxrl} &= -k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pxll} &= -k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pxr} &= k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pxl} &= k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pxr} &= k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pxl} &= k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pybr} &= k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pyll} &= k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pyrl} &= k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pyll} &= k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pyr} &= k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pyl} &= k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pyr} &= k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pyl} &= k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pyr} &= k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pyl} &= k_{p}(h_{p}q_{1}) + c_{p}h_{p}q_{1}; \\
F_{pyr} &= k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2}; \\
F_{pyl} &= k_{p}(h_{p}q_{2}) + c_{p}h_{p}q_{2};
\end{align*}
\]

\[
F_{pz1r} = -k_{p}(\dot{z}_{1} - b_{p}\dot{q}_{1} + d_{p}\dot{\theta}_{1} - \frac{dp}{a}\dot{z}_{1r}) \\
- c_{p}(\ddot{z}_{1} - b_{p}\dot{q}_{1} + d_{p}\dot{\theta}_{1} - \frac{dp}{a}\dot{z}_{1r}); \\
F_{pz1l} = -k_{p}(\dot{z}_{1} - b_{p}\dot{q}_{1} - d_{p}\dot{\theta}_{1} - \frac{dp}{a}\dot{z}_{1l}) \\
- c_{p}(\ddot{z}_{1} - b_{p}\dot{q}_{1} - d_{p}\dot{\theta}_{1} - \frac{dp}{a}\dot{z}_{1l}); \\
F_{pz2r} = -k_{p}(\dot{z}_{1} + b_{p}\dot{q}_{1} - d_{p}\dot{\theta}_{1} - \frac{dp}{a}\dot{z}_{2r}) \\
- c_{p}(\ddot{z}_{1} + b_{p}\dot{q}_{1} - d_{p}\dot{\theta}_{1} - \frac{dp}{a}\dot{z}_{2r}); \\
F_{pz2l} = -k_{p}(\dot{z}_{1} + b_{p}\dot{q}_{1} - d_{p}\dot{\theta}_{1} - \frac{dp}{a}\dot{z}_{2l}) \\
- c_{p}(\ddot{z}_{1} + b_{p}\dot{q}_{1} - d_{p}\dot{\theta}_{1} - \frac{dp}{a}\dot{z}_{2l}); \\
F_{pz3r} = -k_{p}(\dot{z}_{2} - b_{p}\dot{q}_{2} + d_{p}\dot{\theta}_{2} - \frac{dp}{a}\dot{z}_{3r}) \\
- c_{p}(\ddot{z}_{2} - b_{p}\dot{q}_{2} + d_{p}\dot{\theta}_{2} - \frac{dp}{a}\dot{z}_{3r}); \\
F_{pz3l} = -k_{p}(\dot{z}_{2} - b_{p}\dot{q}_{2} + d_{p}\dot{\theta}_{2} - \frac{dp}{a}\dot{z}_{3l}) \\
- c_{p}(\ddot{z}_{2} - b_{p}\dot{q}_{2} + d_{p}\dot{\theta}_{2} - \frac{dp}{a}\dot{z}_{3l}); \\
F_{pz4r} = -k_{p}(\dot{z}_{2} + b_{p}\dot{q}_{2} + d_{p}\dot{\theta}_{2} - \frac{dp}{a}\dot{z}_{4r}) \\
- c_{p}(\ddot{z}_{2} + b_{p}\dot{q}_{2} + d_{p}\dot{\theta}_{2} - \frac{dp}{a}\dot{z}_{4r}); \\
F_{pz4l} = -k_{p}(\dot{z}_{2} + b_{p}\dot{q}_{2} - d_{p}\dot{\theta}_{2} - \frac{dp}{a}\dot{z}_{4l}) \\
- c_{p}(\ddot{z}_{2} + b_{p}\dot{q}_{2} - d_{p}\dot{\theta}_{2} - \frac{dp}{a}\dot{z}_{4l});
\]

REFERENCES


